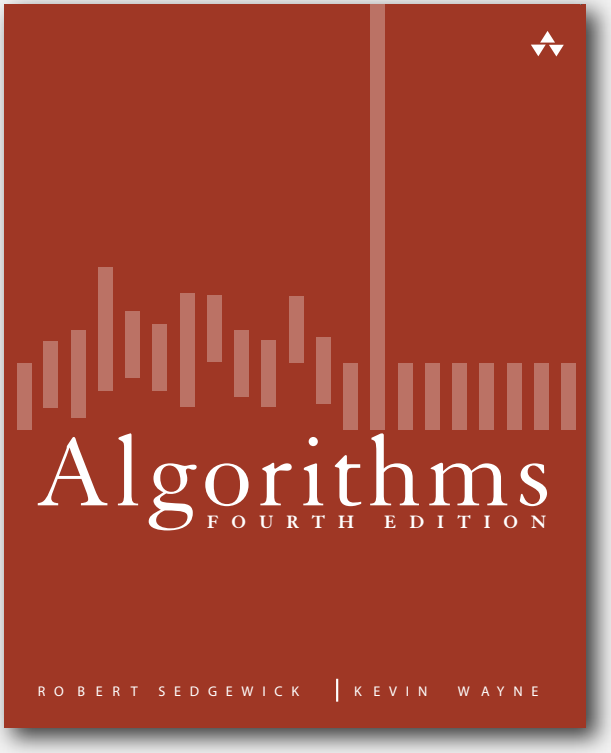
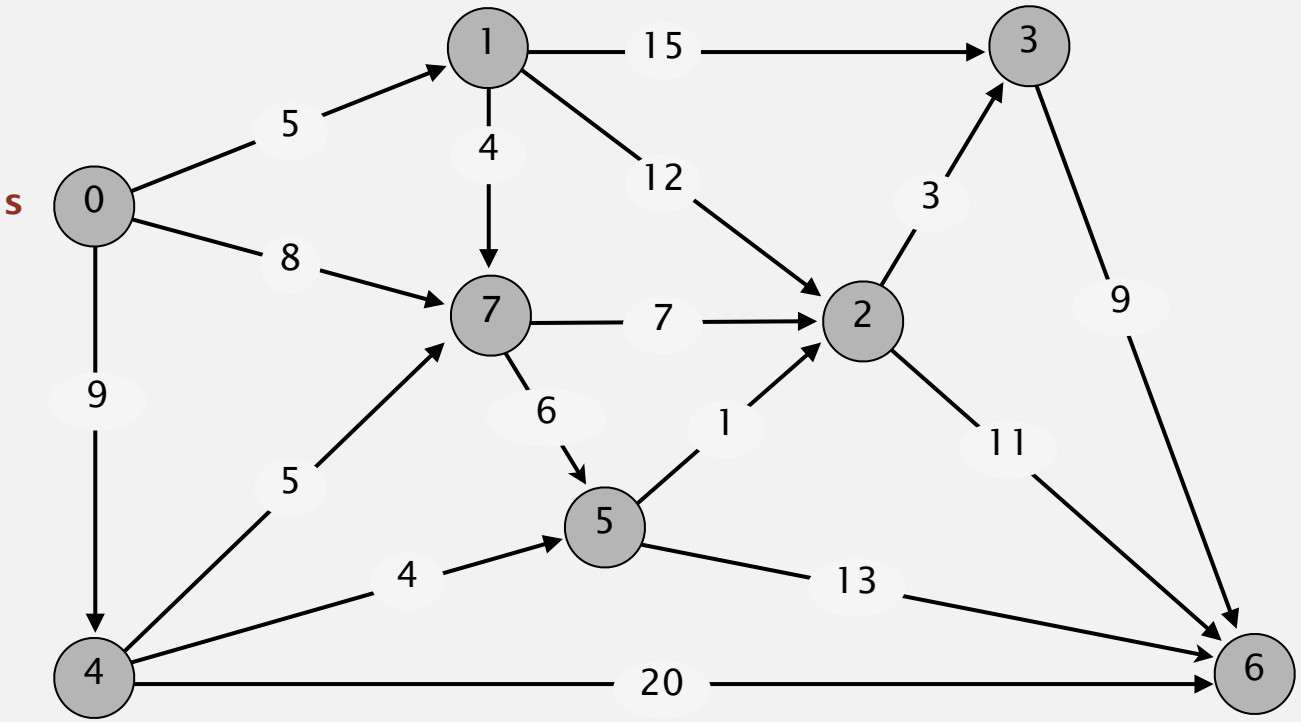
4.4 Acyclic Shortest Paths Demo



Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

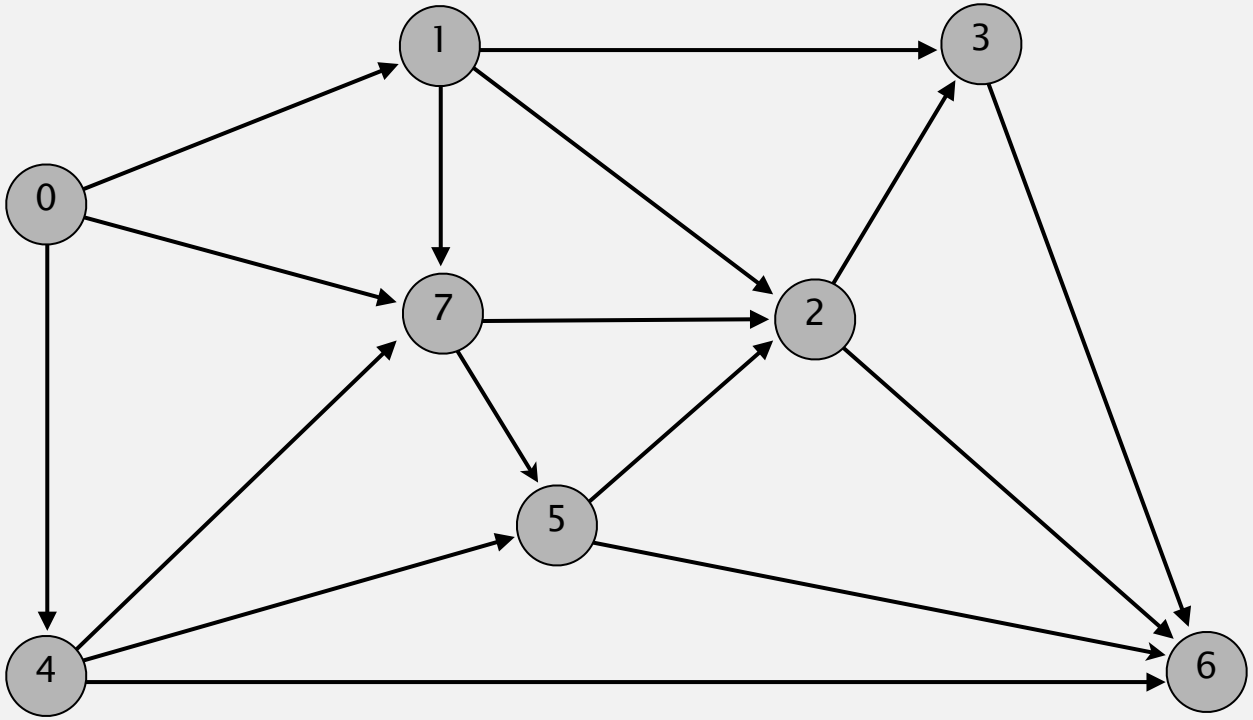


an edge-weighted DAG

```
0→1   5.0
0→4   9.0
0→7   8.0
1→2  12.0
1→3  15.0
1→7   4.0
2→3   3.0
2→6  11.0
3→6   9.0
4→5   4.0
4→6  20.0
4→7   5.0
5→2   1.0
5→6  13.0
7→5   6.0
7→2   7.0
```

Topological sort algorithm

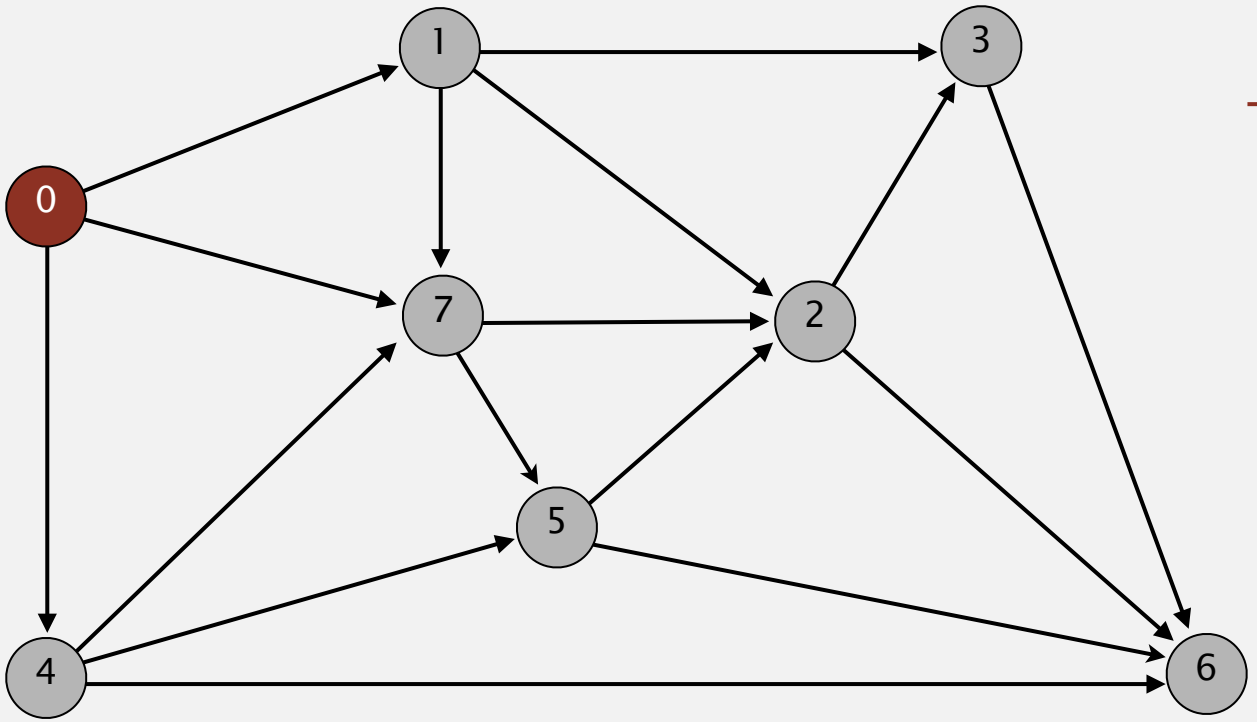
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



topological order: 0 1 4 7 5 2 3 6

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



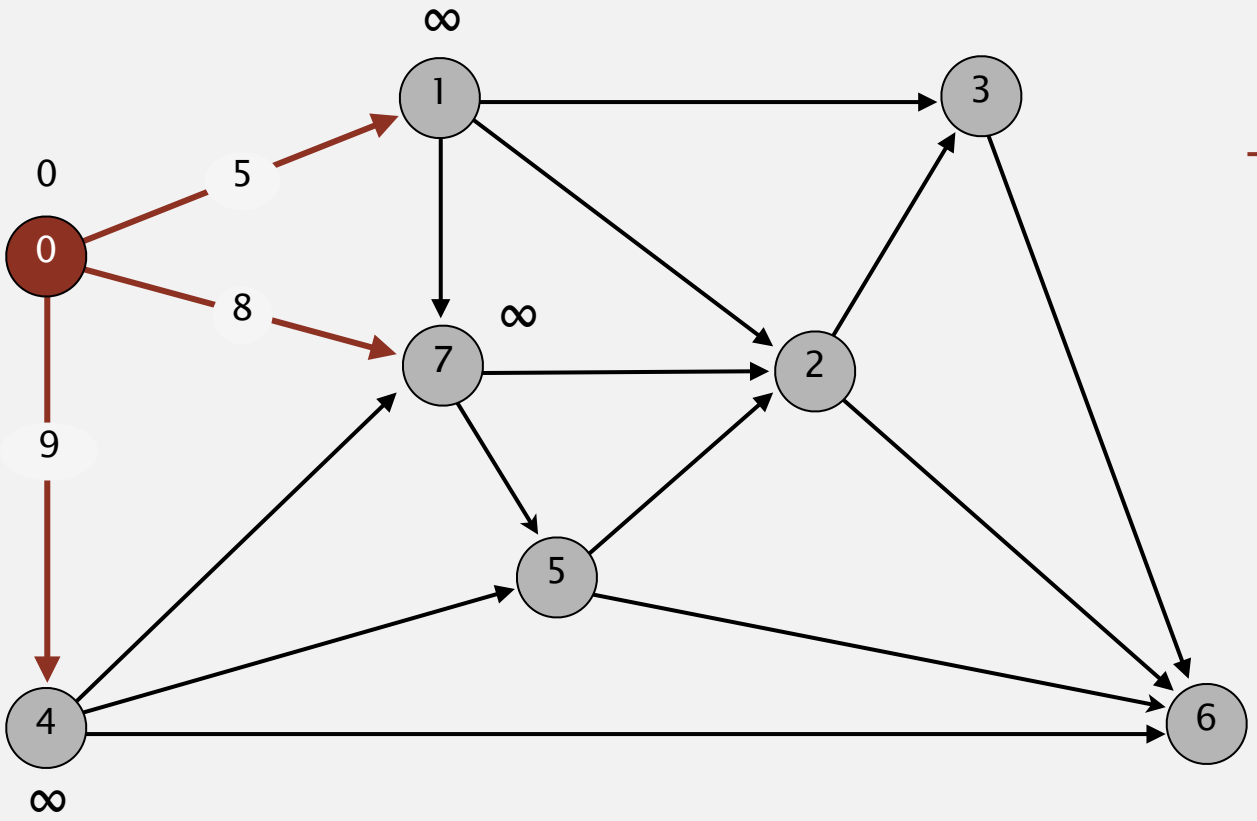
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1		
2		
3		
4		
5		
6		
7		

choose vertex 0

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



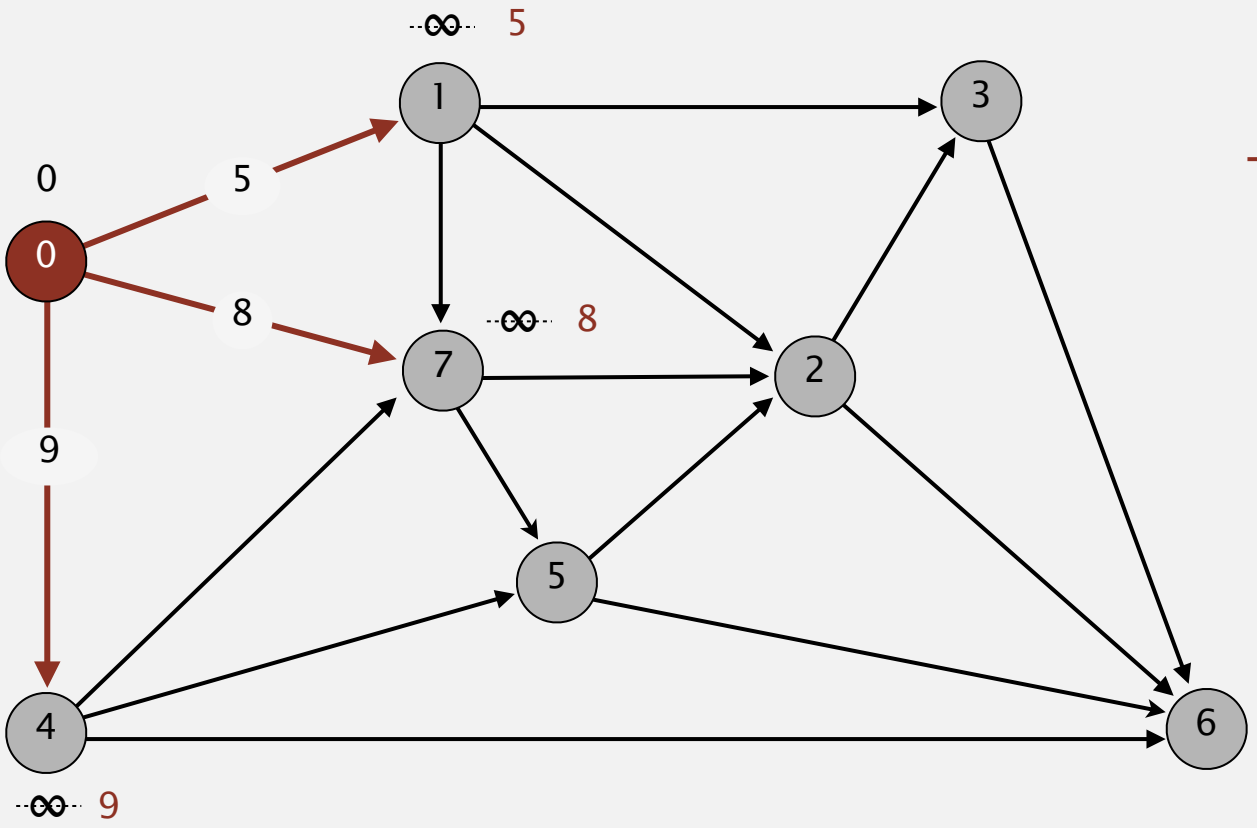
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1		
2		
3		
4		
5		
6		
7		

relax all edges incident from 0

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



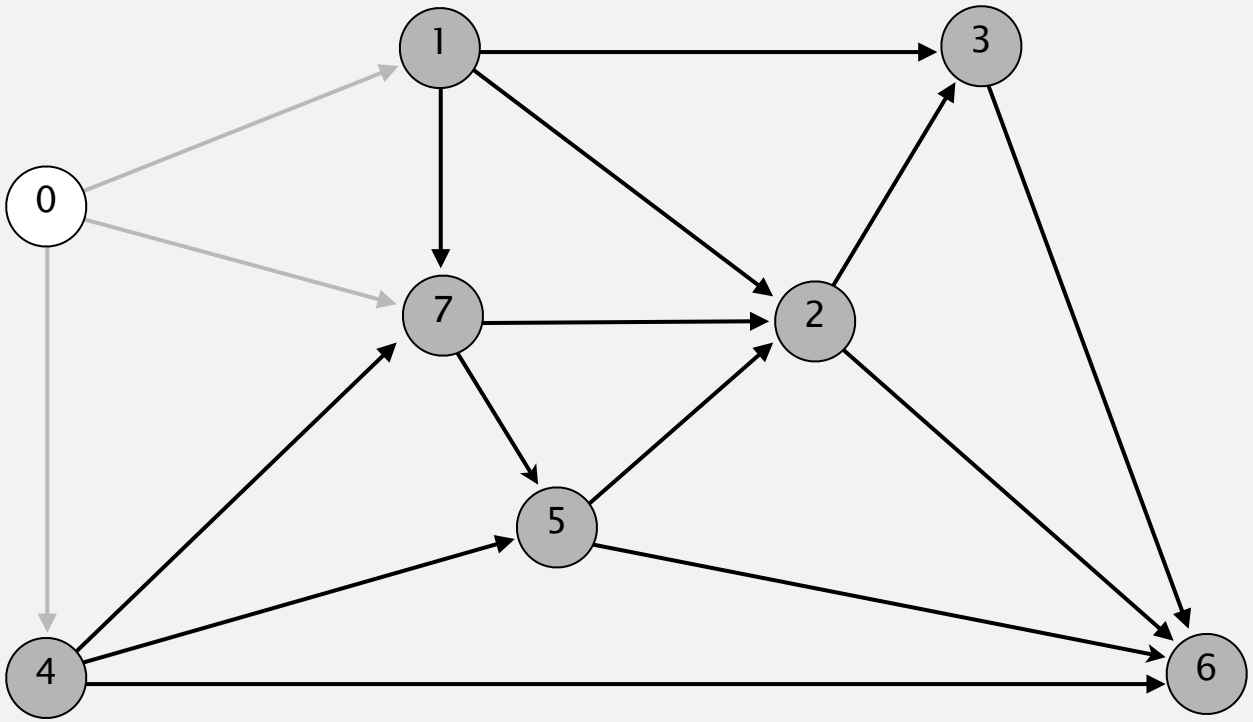
0 1 4 7 5 2 3 6

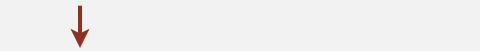
v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2		
3		
4	9.0	0→4
5		
6		
7	8.0	0→7

relax all edges incident from 0

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

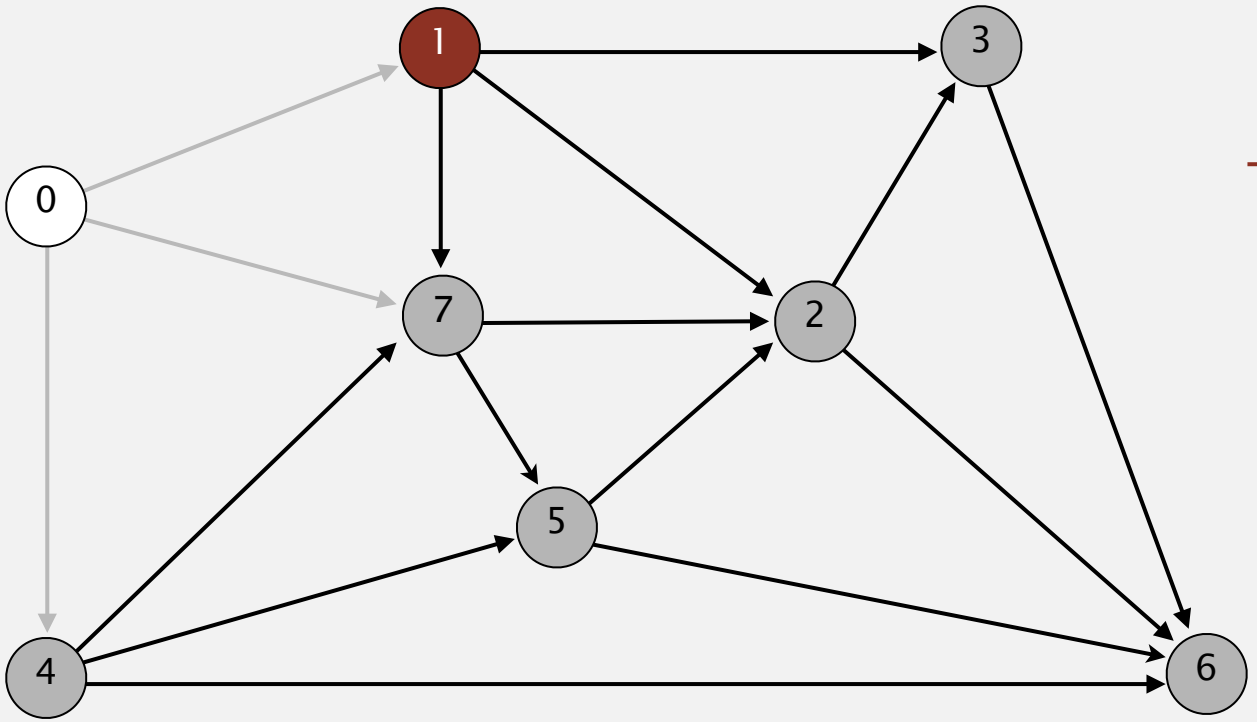




v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2		
3		
4	9.0	0→4
5		
6		
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



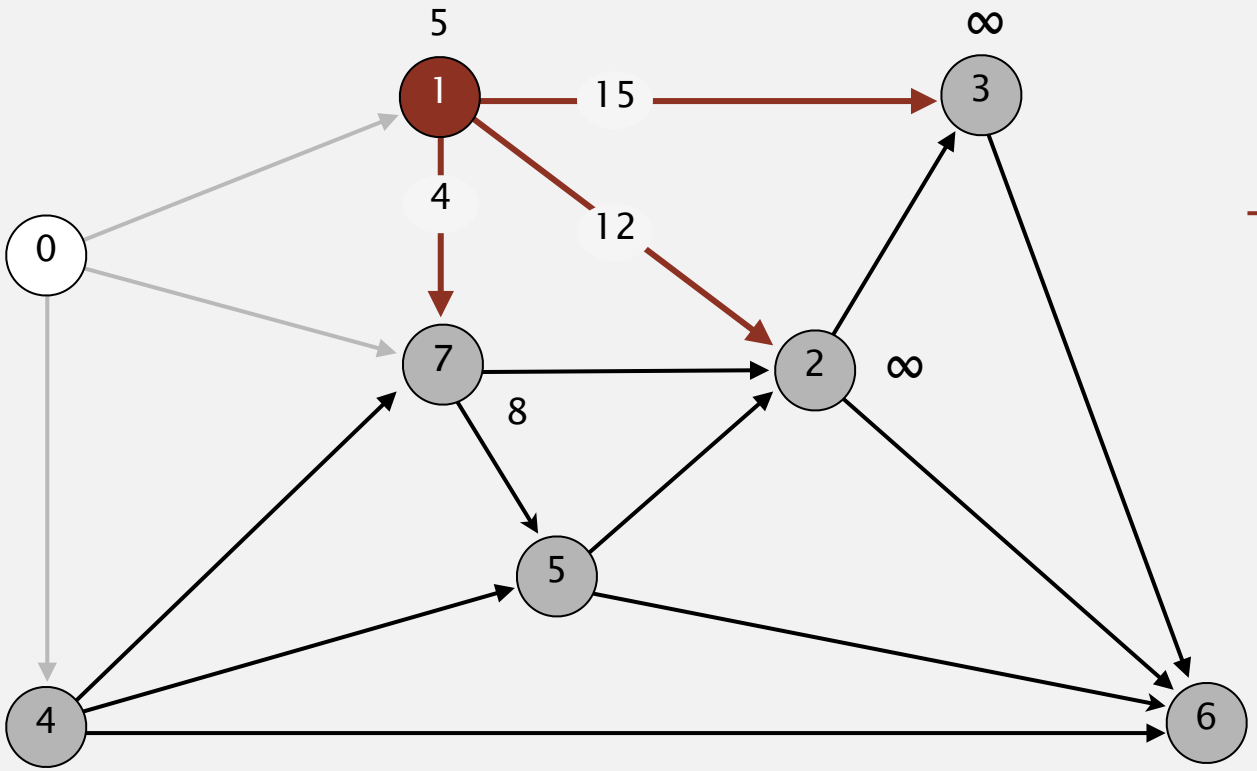
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
→ 1	5.0	0→1
2		
3		
4	9.0	0→4
5		
6		
7	8.0	0→7

choose vertex 1

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



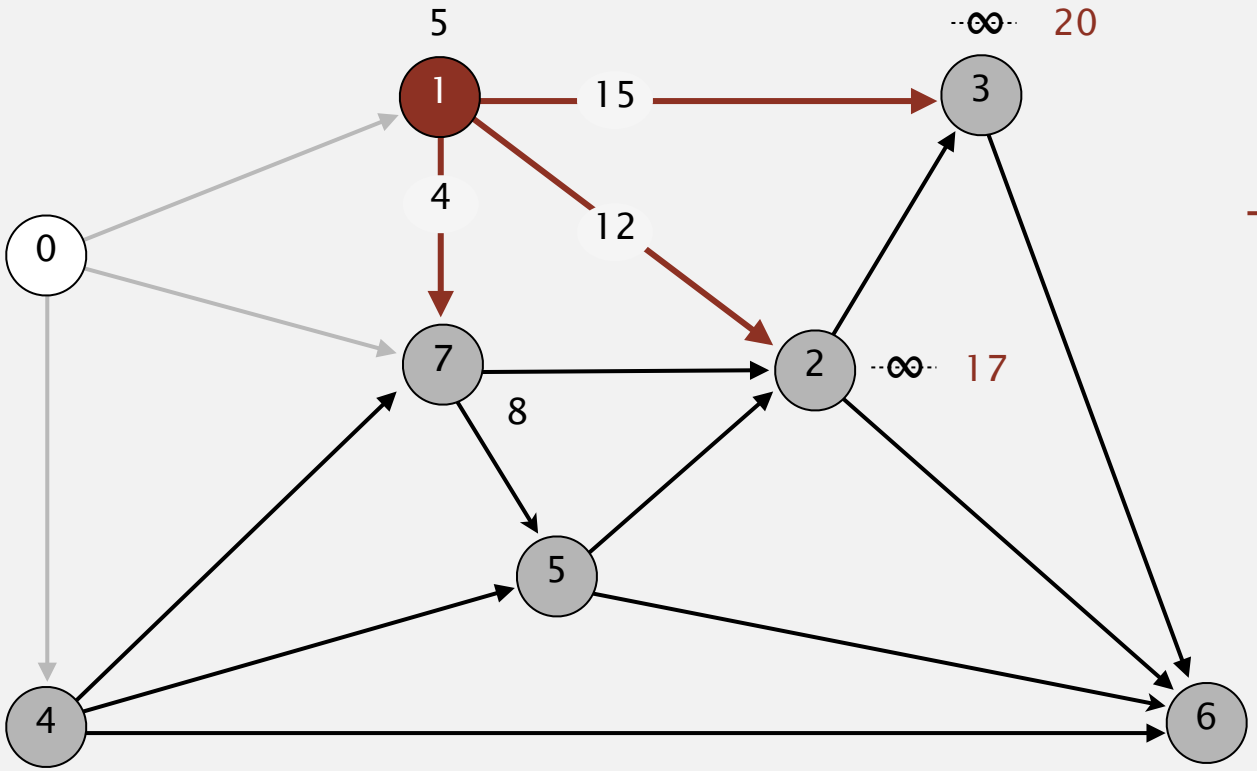
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
→ 1	5.0	0→1
2		
3		
4	9.0	0→4
5		
6		
7	8.0	0→7

relax all edges incident from 1

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



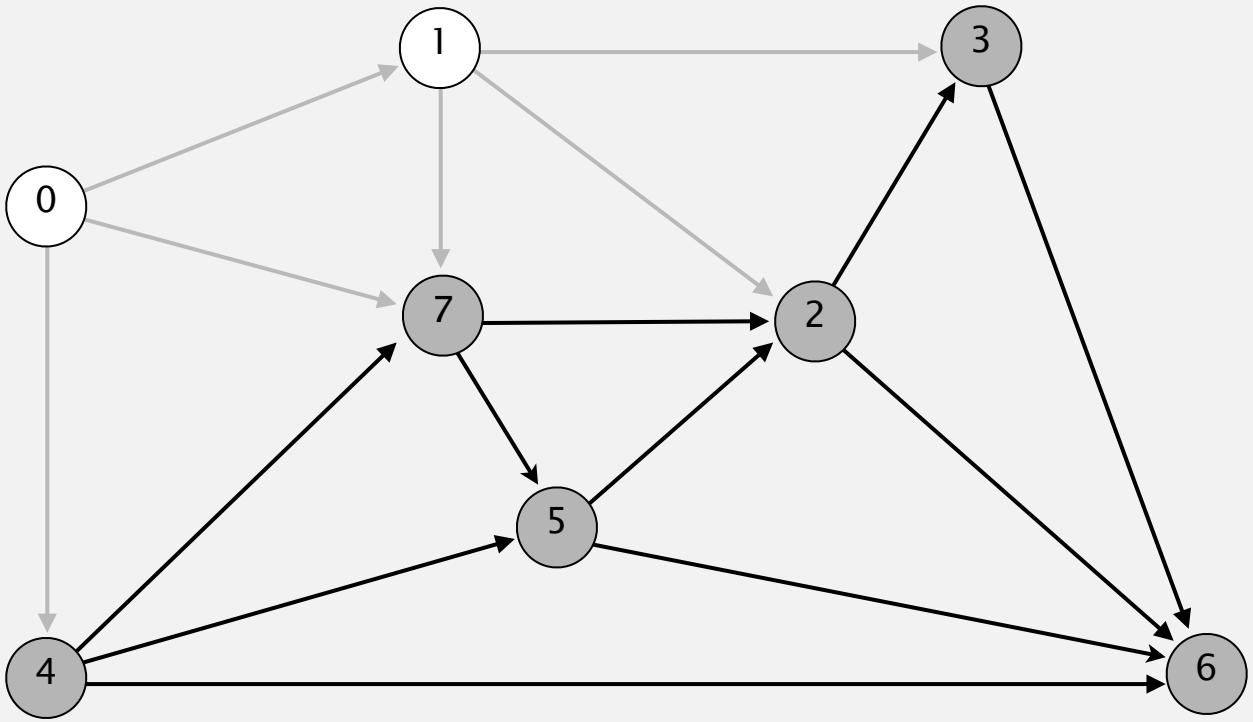
0 1 4 7 5 2 3 6

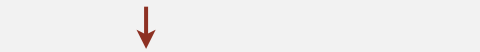
v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
4	9.0	0→4
5		
6		
7	8.0 ✔	0→7

relax all edges incident from 1

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

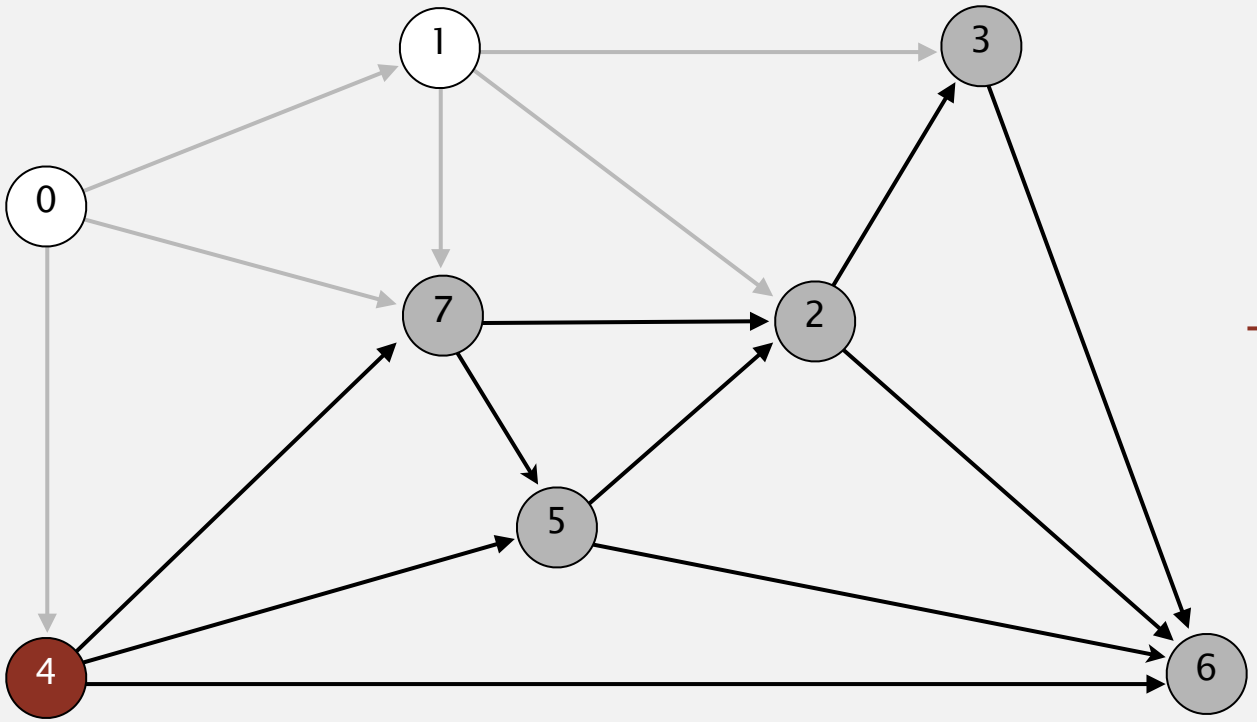




v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
4	9.0	0→4
5		
6		
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



0 1 4 7 5 2 3 6

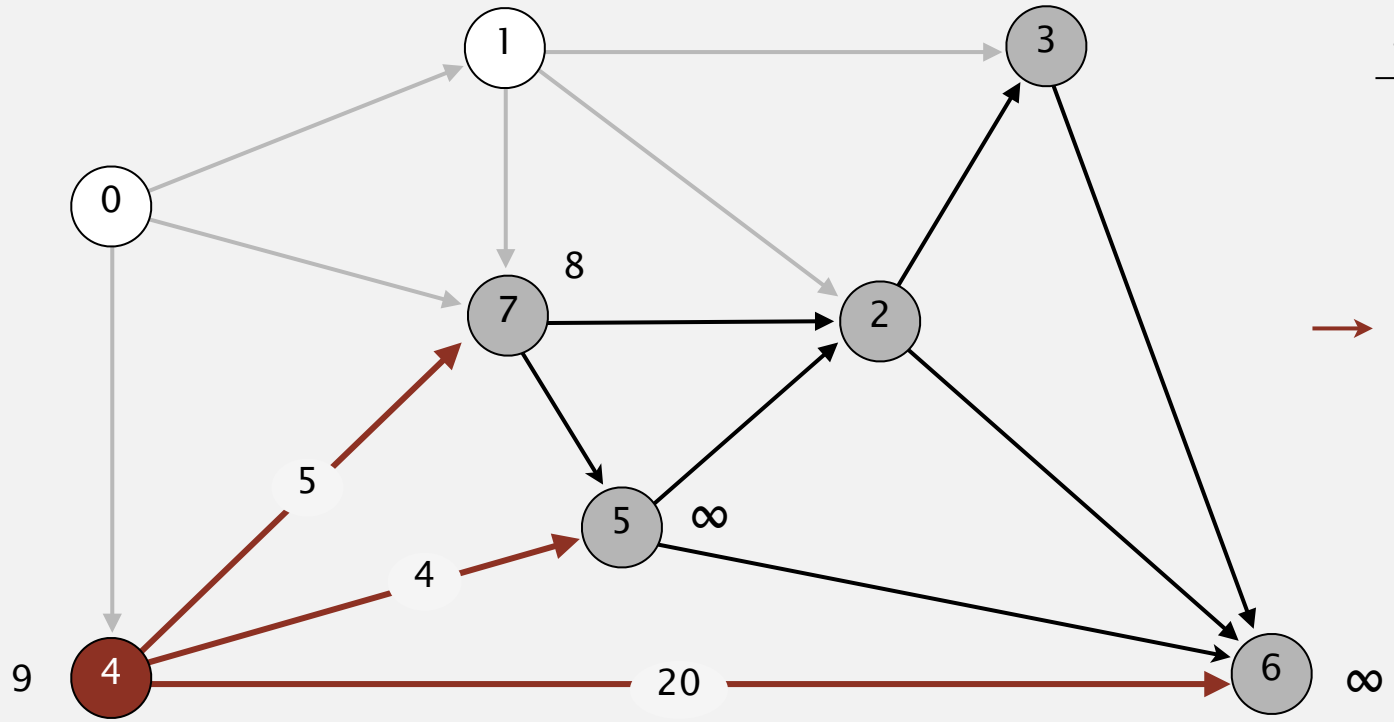
v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
→ 4	9.0	0→4
5		
6		
7	8.0	0→7

select vertex 4

(Dijkstra would have selected vertex 7)

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



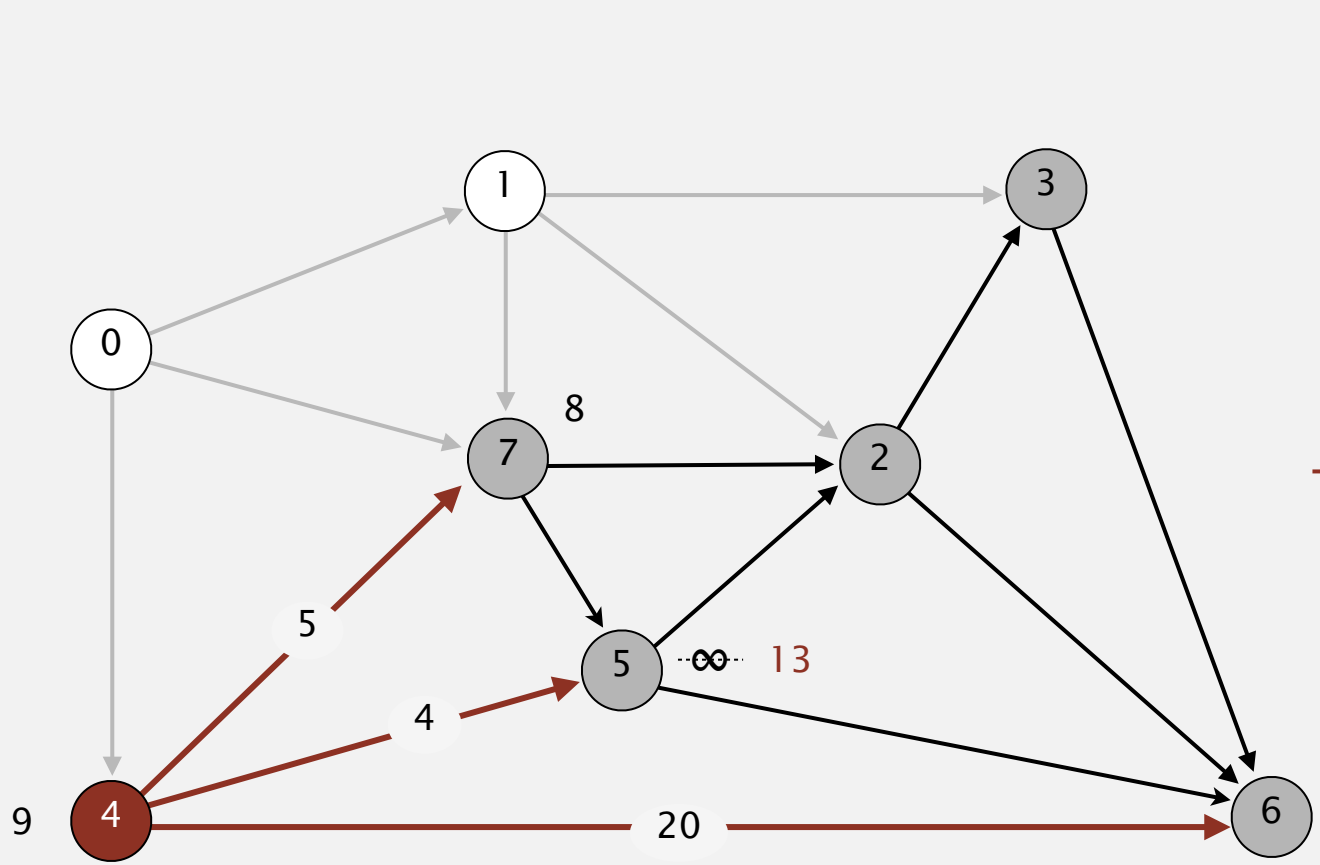
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
→ 4	9.0	0→4
5		
6		
7	8.0	0→7

relax all edges incident from 4

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



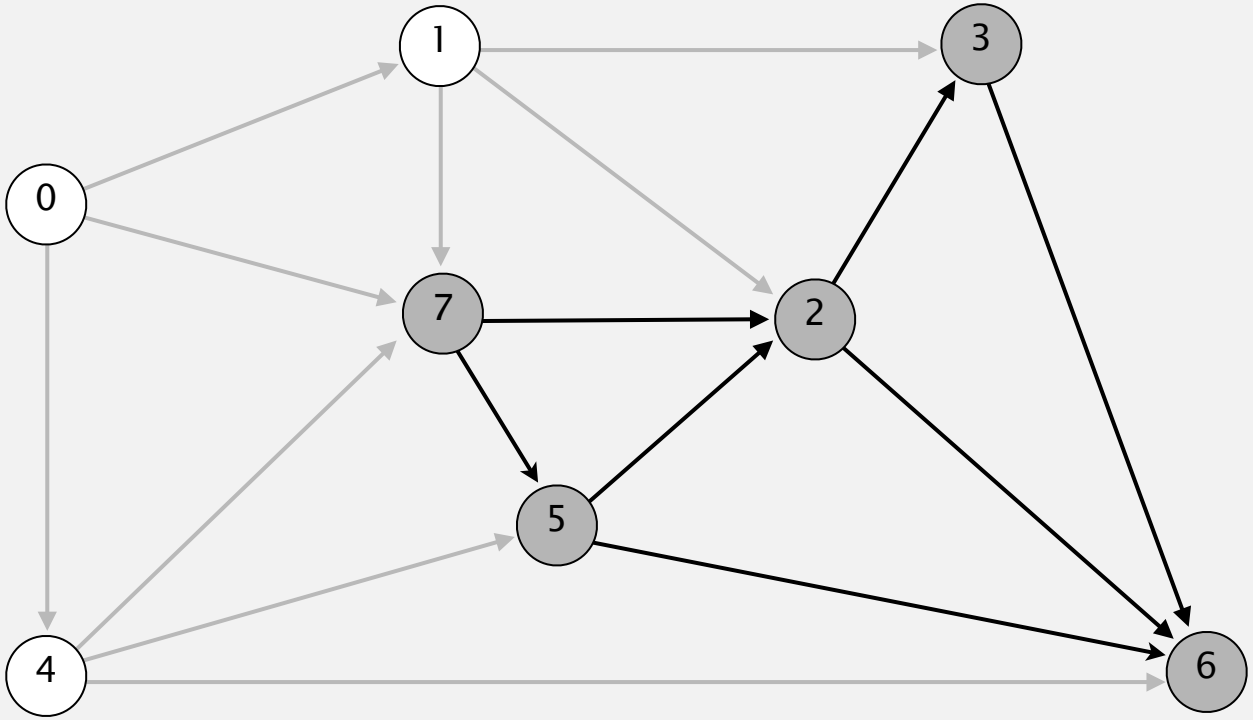
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
→ 4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
7	8.0 ✔	0→7

relax all edges incident from 4

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

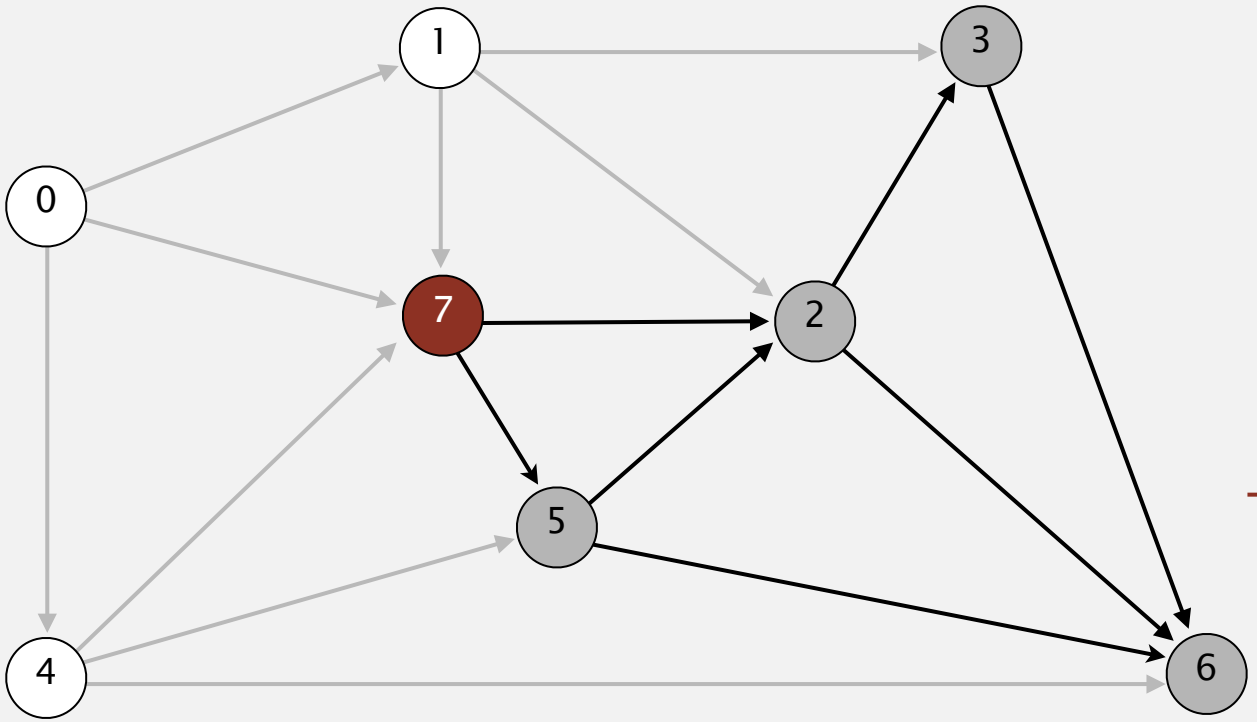


0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



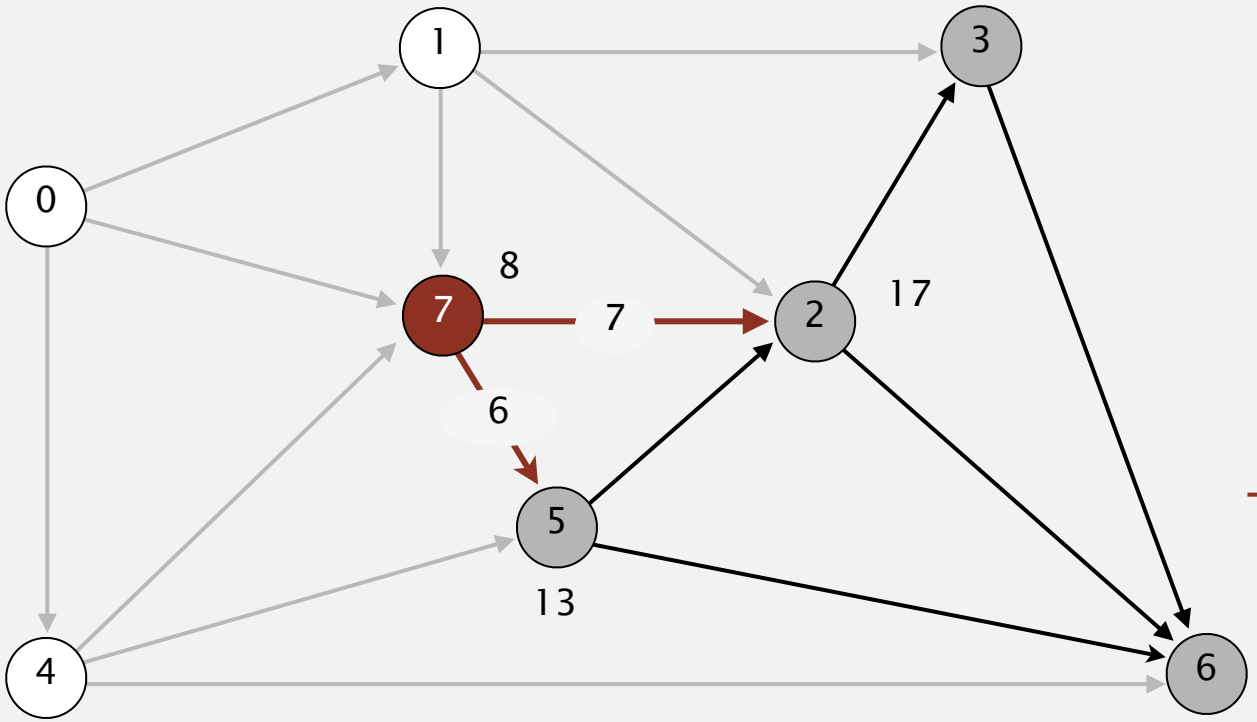
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
→ 7	8.0	0→7

choose vertex 7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



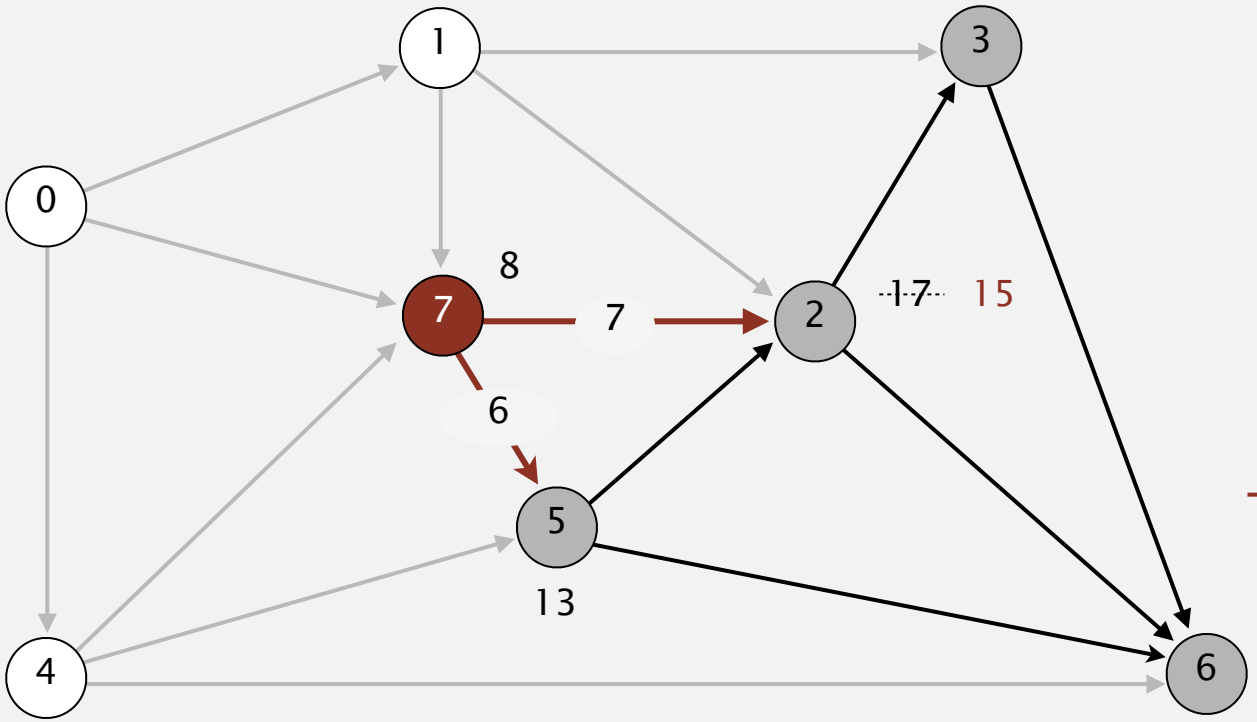
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	17.0	1→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
→ 7	8.0	0→7

relax all edges incident from 7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



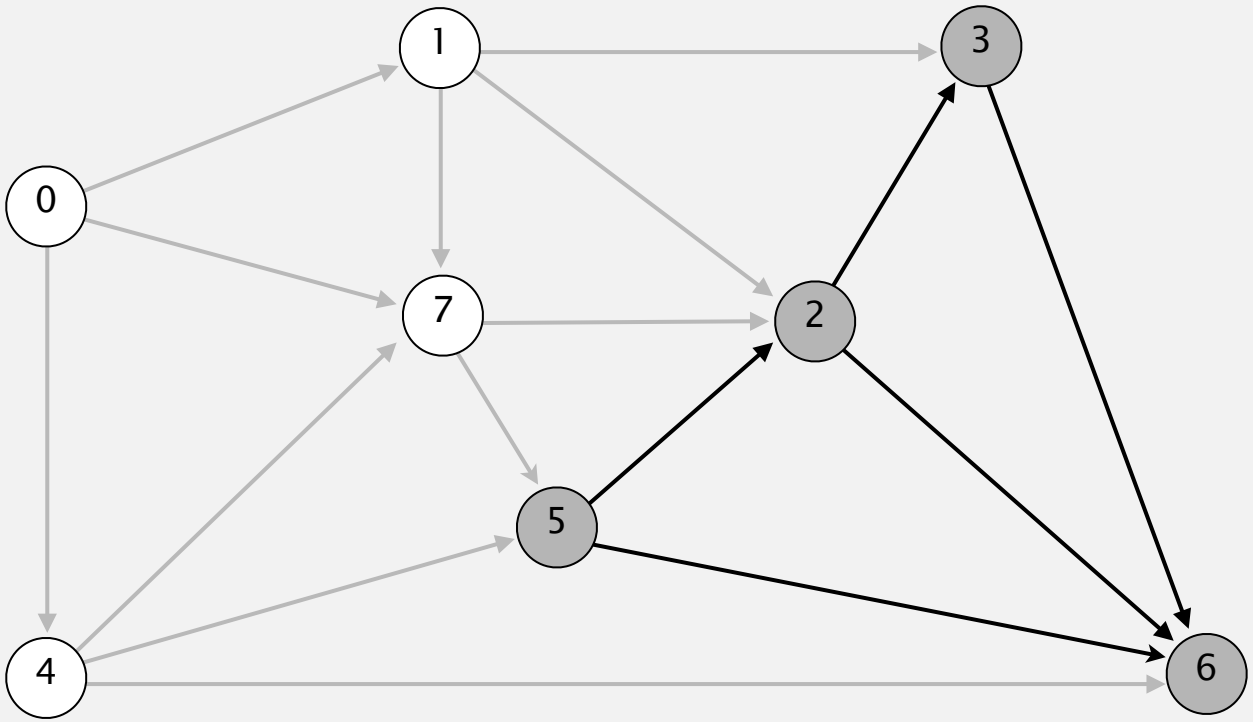
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	15.0	7→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
→ 7	8.0	0→7

relax all edges incident from 7

Topological sort algorithm

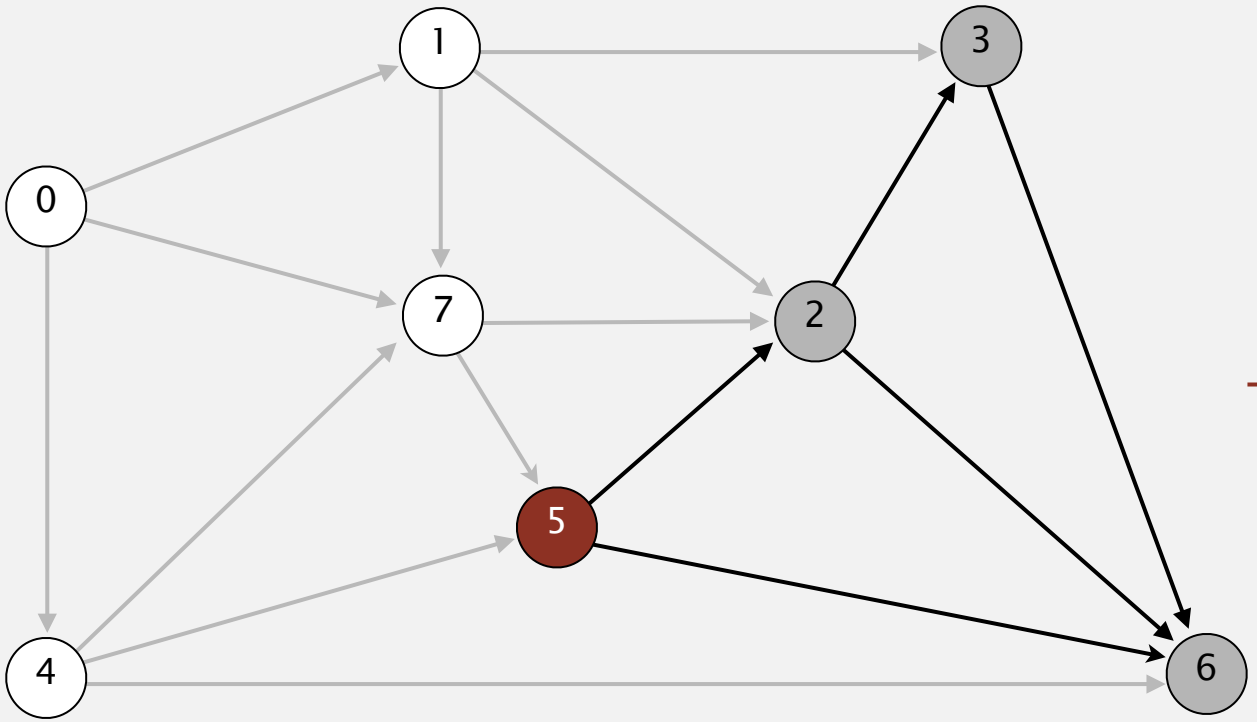
- Consider vertices in topological order.
- Relax all edges incident from that vertex.



v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	15.0	7→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	29.0	4→6
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



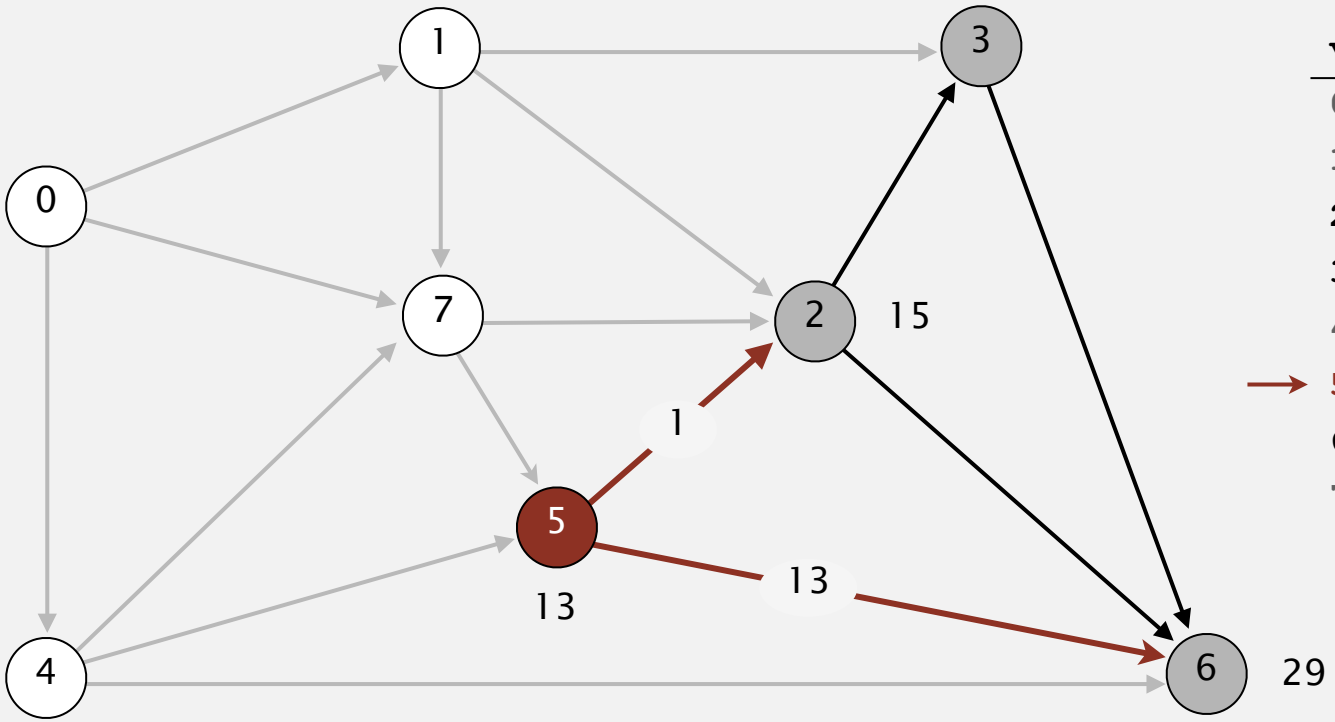
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	15.0	7→2
3	20.0	1→3
4	9.0	0→4
→ 5	13.0	4→5
6	29.0	4→6
7	8.0	0→7

select vertex 5

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



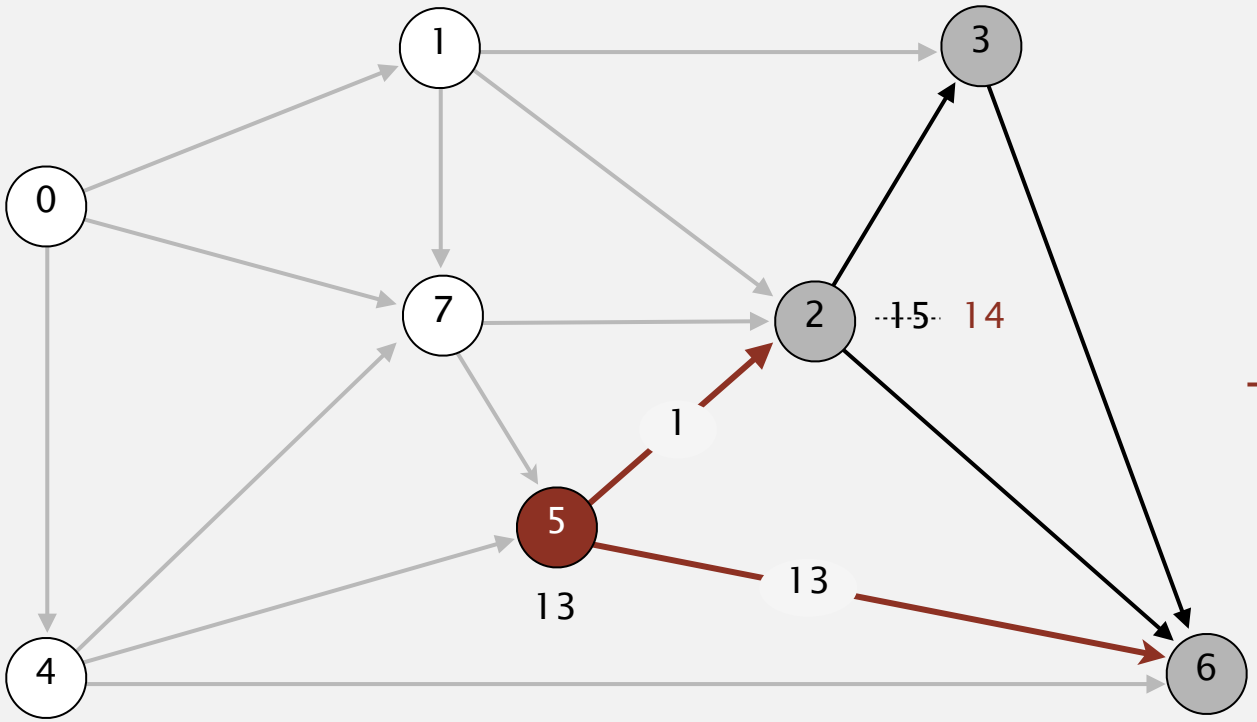
0 1 4 7 **5** 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	15.0	7→2
3	20.0	1→3
4	9.0	0→4
→ 5	13.0	4→5
6	29.0	4→6
7	8.0	0→7

relax all edges incident from 5

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



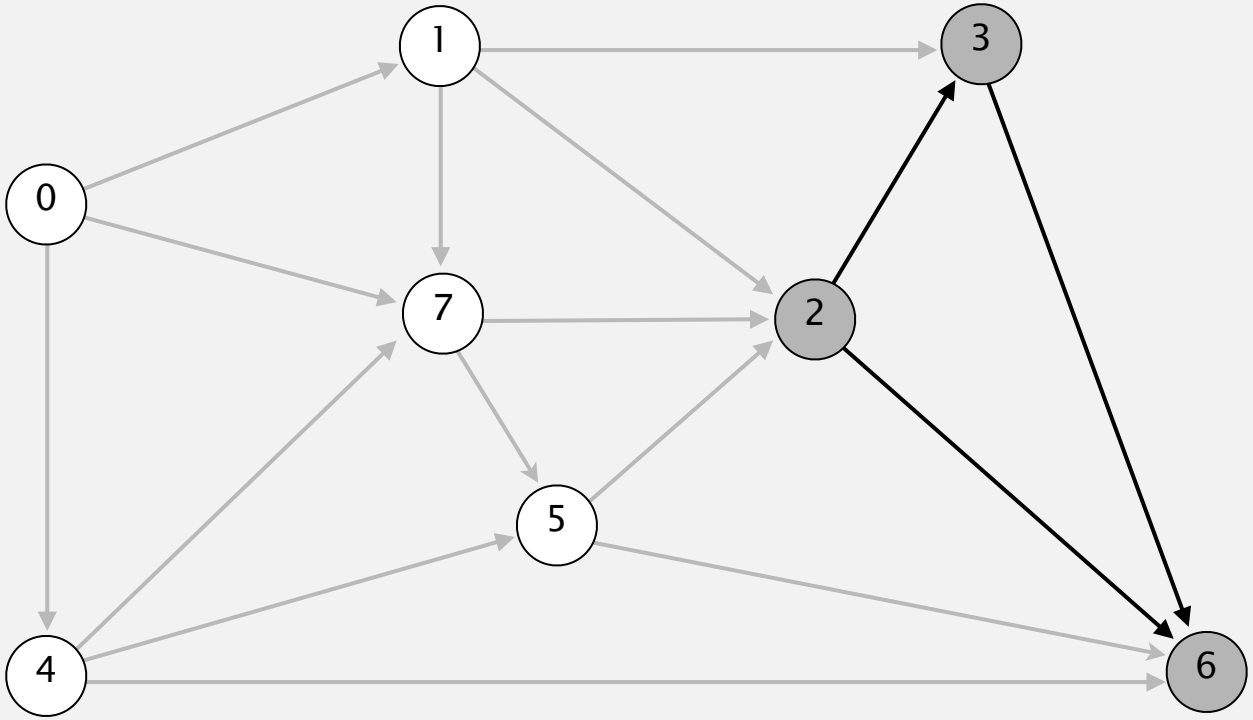
0 1 4 7 **5** 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	20.0	1→3
4	9.0	0→4
→ 5	13.0	4→5
6	26.0	5→6
7	8.0	0→7

relax all edges incident from 5

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

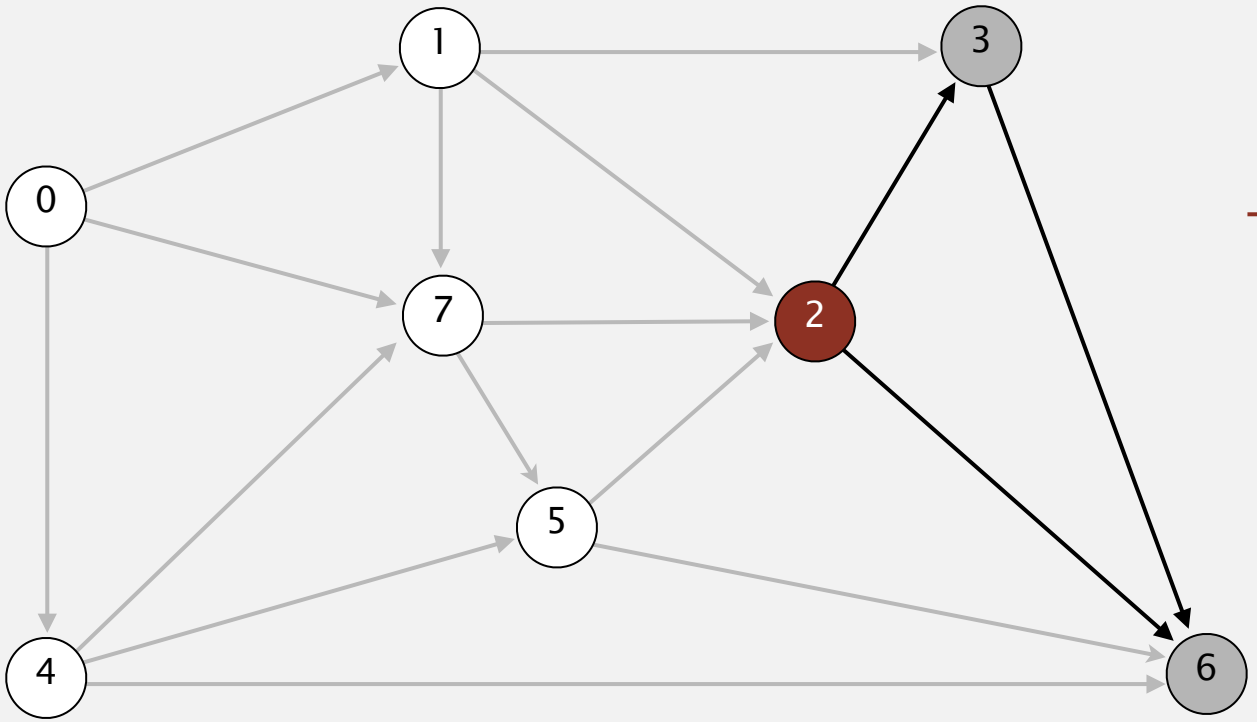


0 1 4 7 5 **2** **3** **6**

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	**14.0**	**5→2**
3	**20.0**	**1→3**
4	9.0	0→4
5	13.0	4→5
6	**26.0**	**5→6**
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



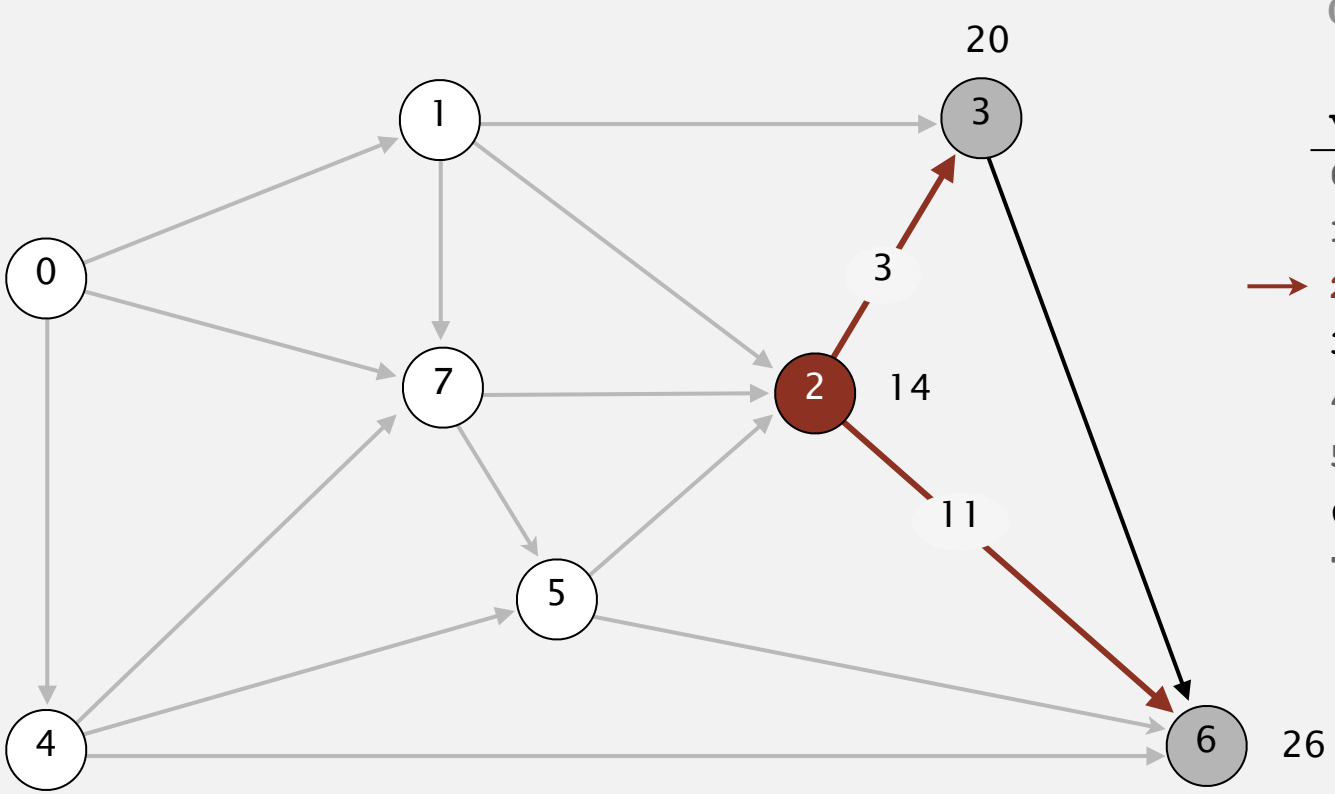
0 1 4 7 5 **2** 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
→ 2	14.0	5→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	26.0	5→6
7	8.0	0→7

select vertex 2

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



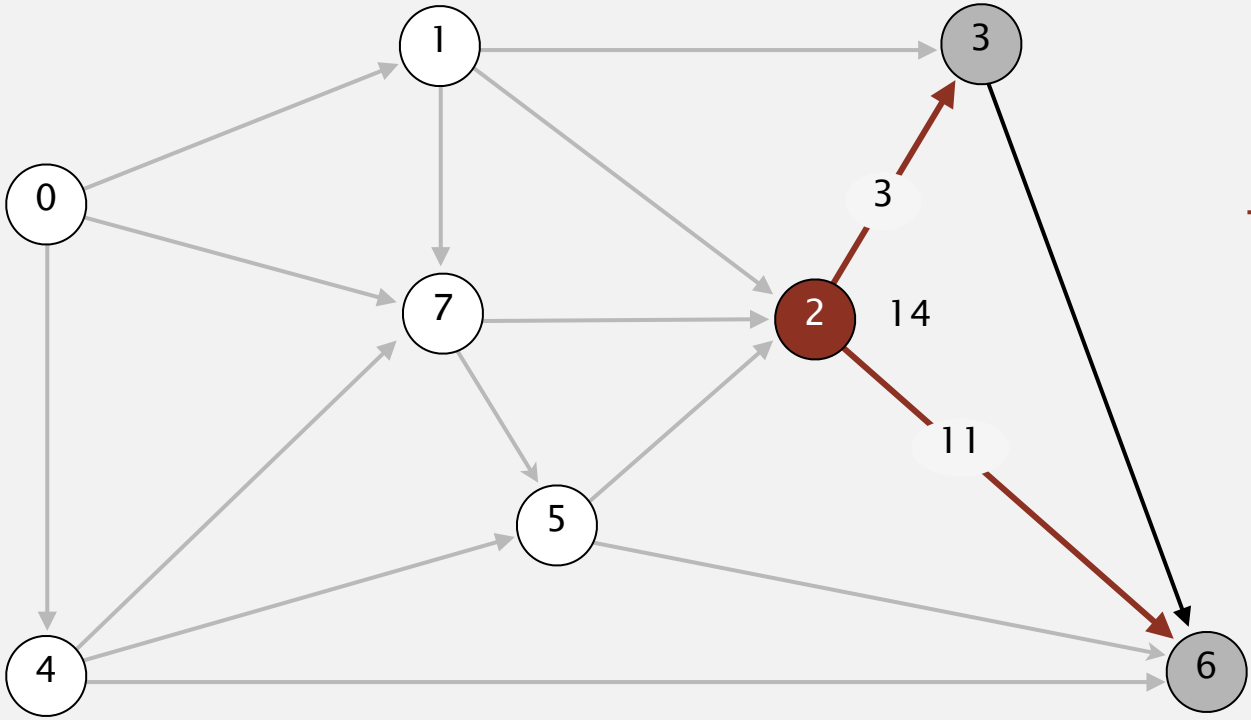
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
→ 2	14.0	5→2
3	20.0	1→3
4	9.0	0→4
5	13.0	4→5
6	26.0	5→6
7	8.0	0→7

relax all edges incident from 2

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



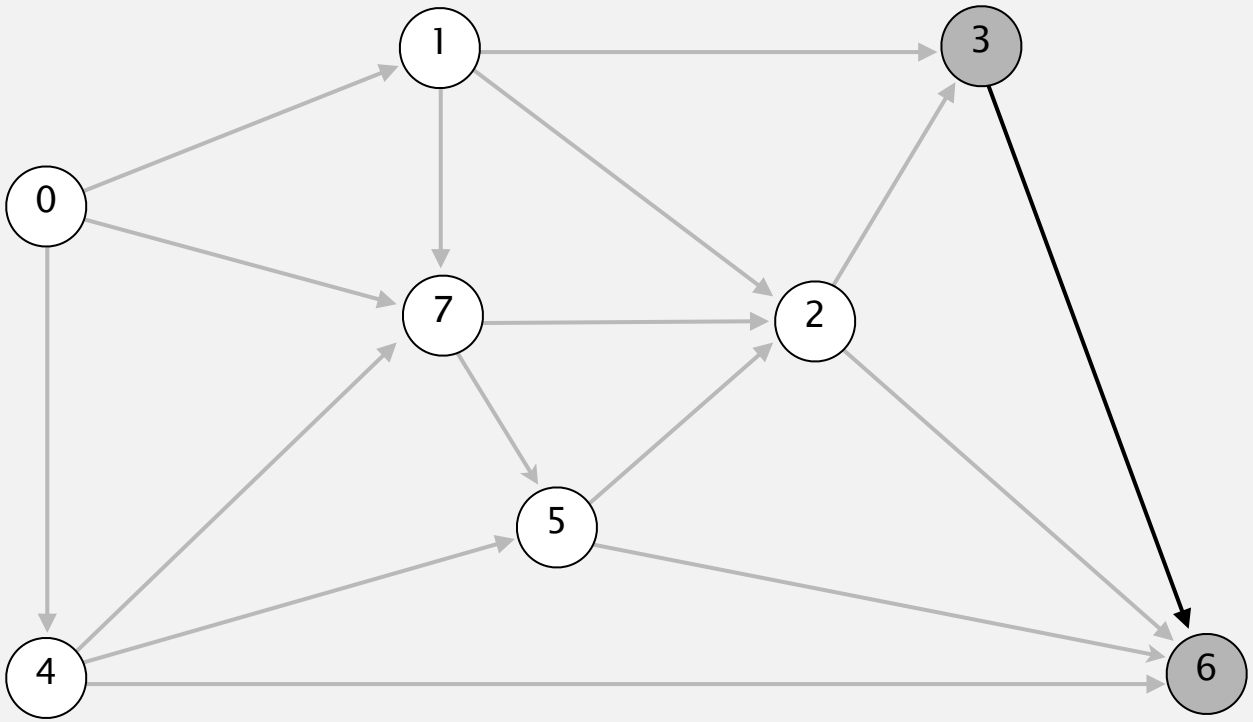
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0	2→6
7	8.0	0→7

relax all edges incident from 2

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

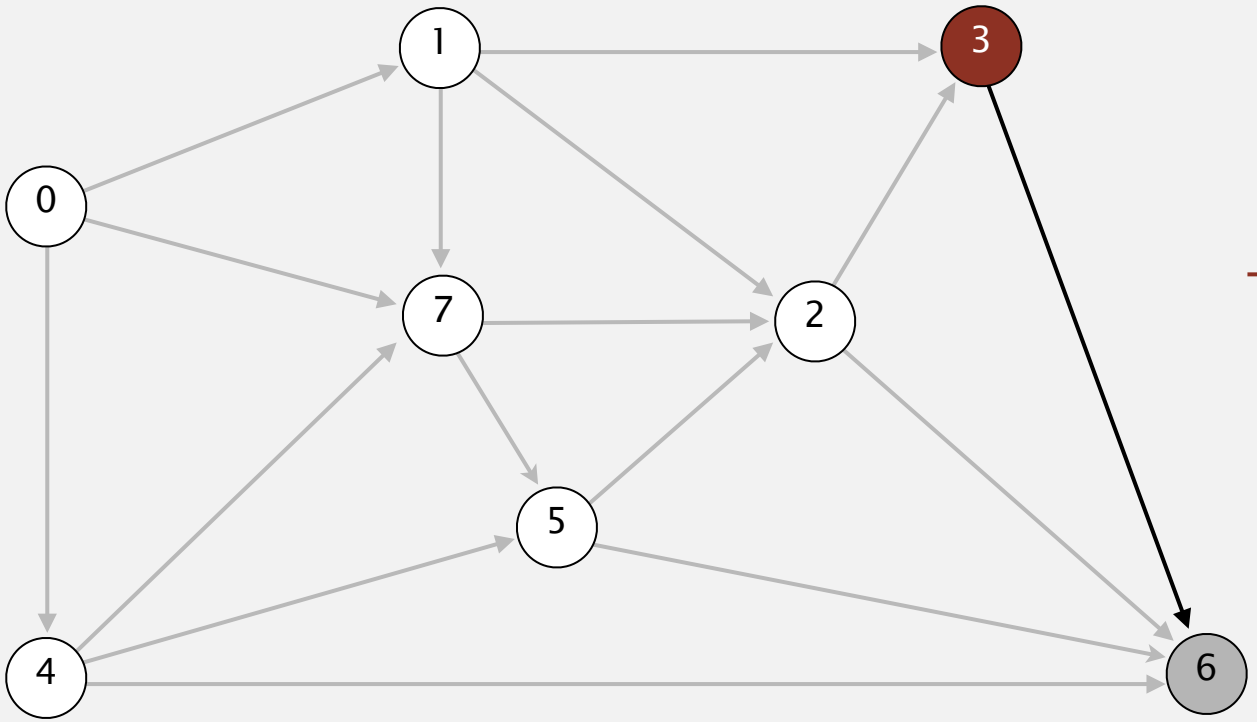


0 1 4 7 5 2 **3** **6**

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	**17.0**	**2→3**
4	9.0	0→4
5	13.0	4→5
6	**25.0**	**2→6**
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



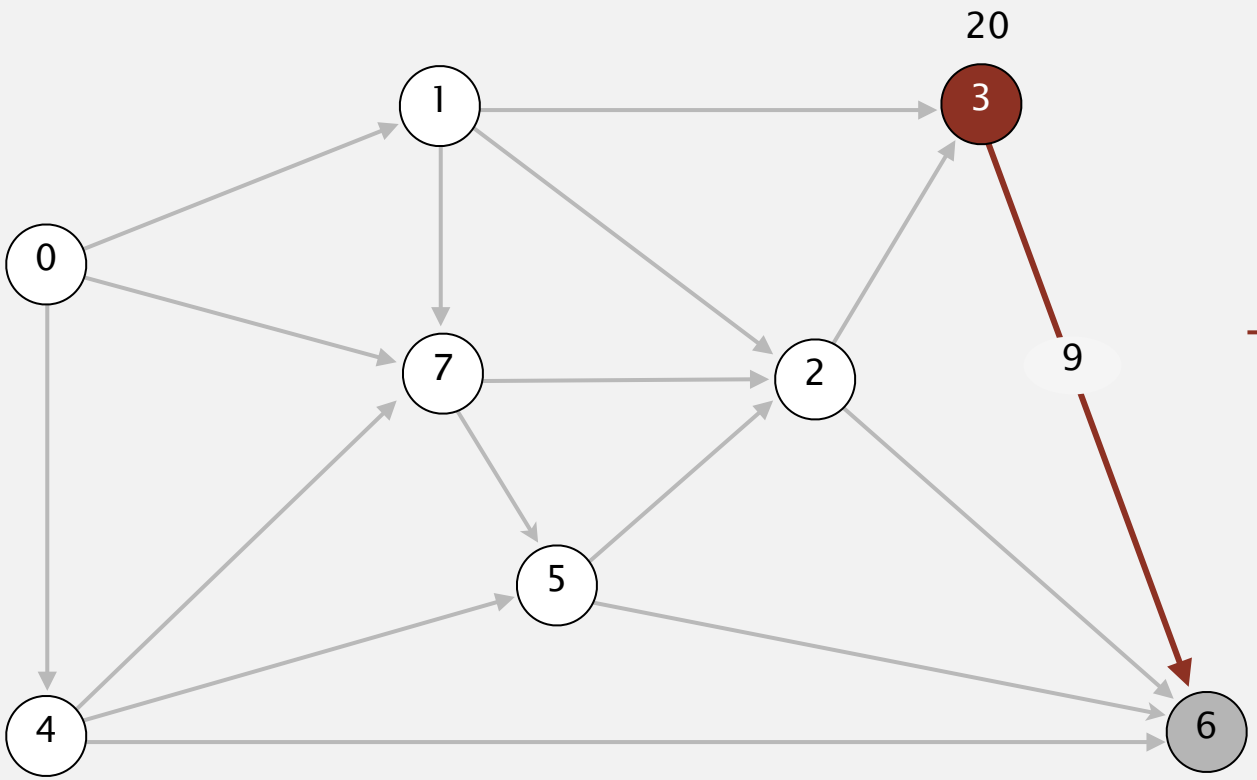
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
→ 3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0	2→6
7	8.0	0→7

select vertex 3

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



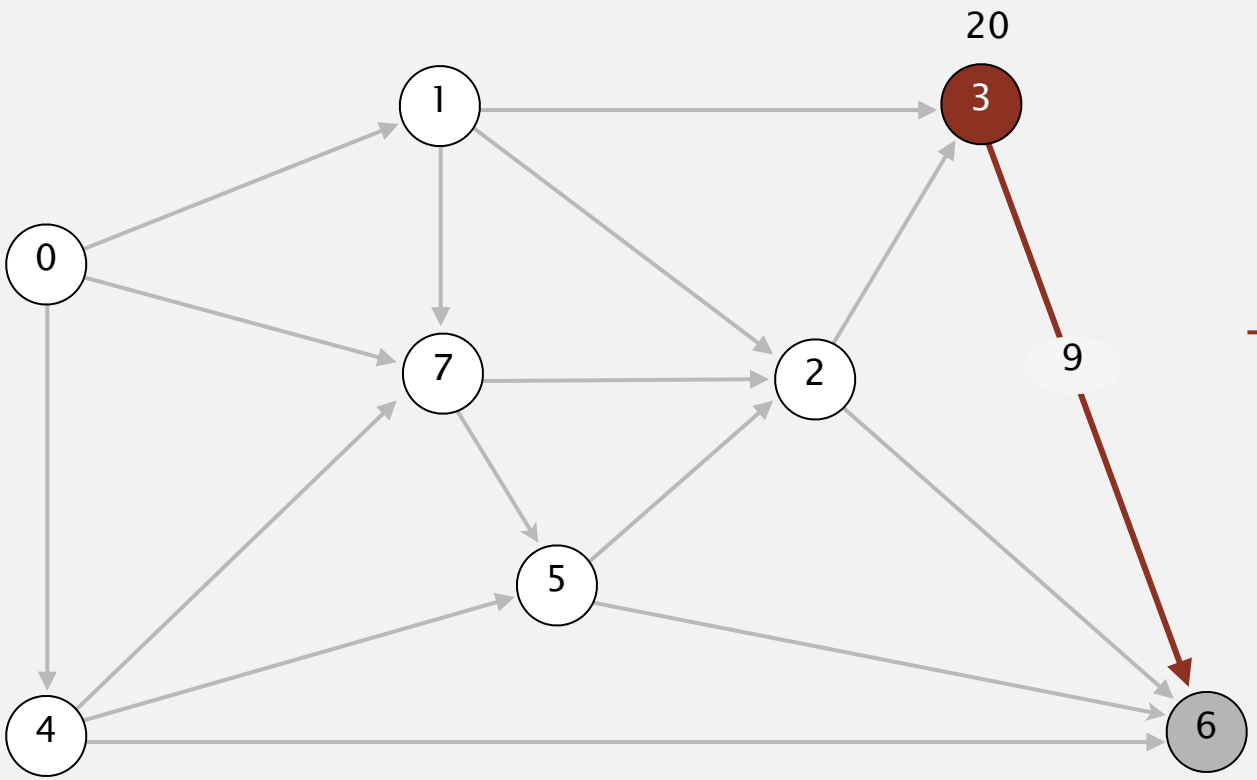
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0	2→6
7	8.0	0→7

relax all edges incident from 3

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



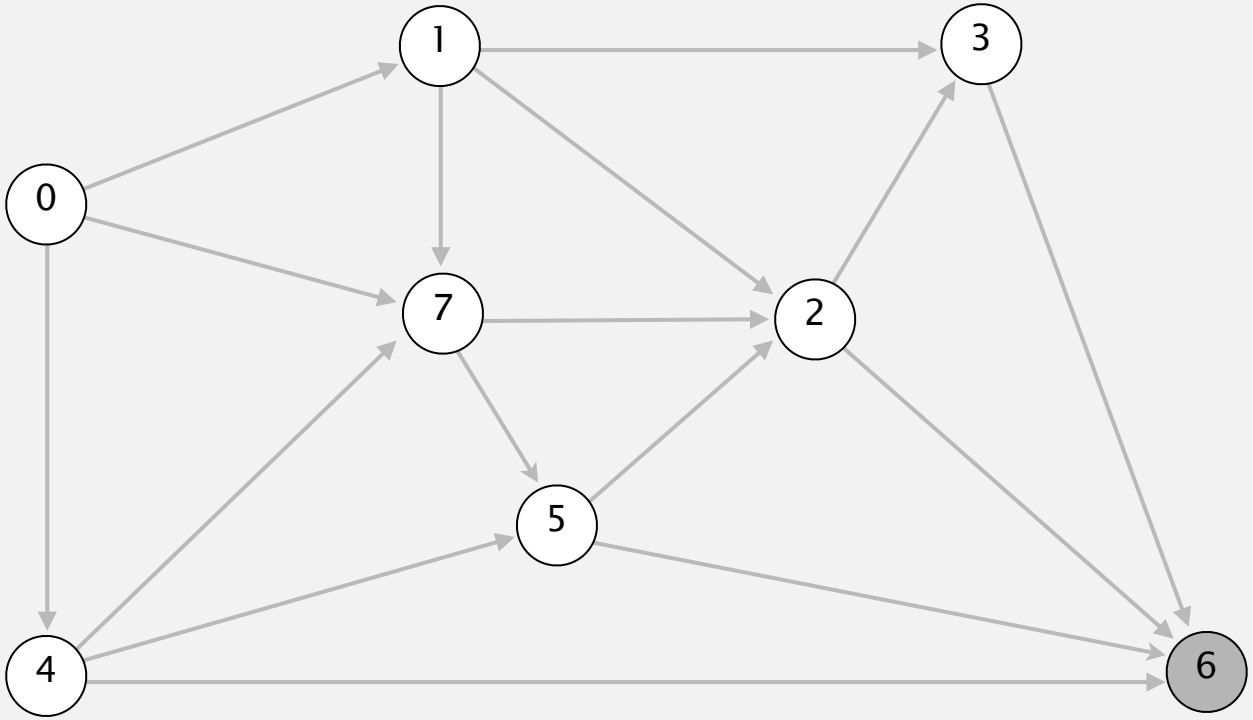
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0 ✔	2→6
7	8.0	0→7

relax all edges incident from 3

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

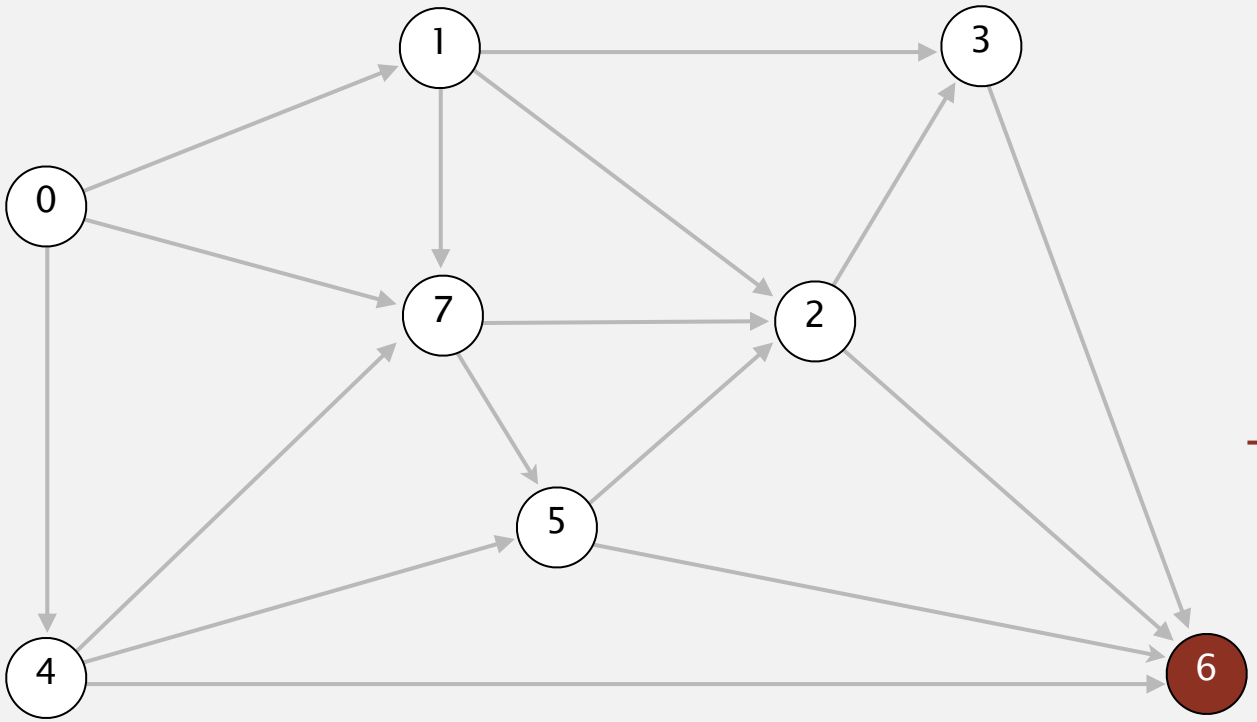


0 1 4 7 5 2 3 **6**

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	**25.0**	**2→6**
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



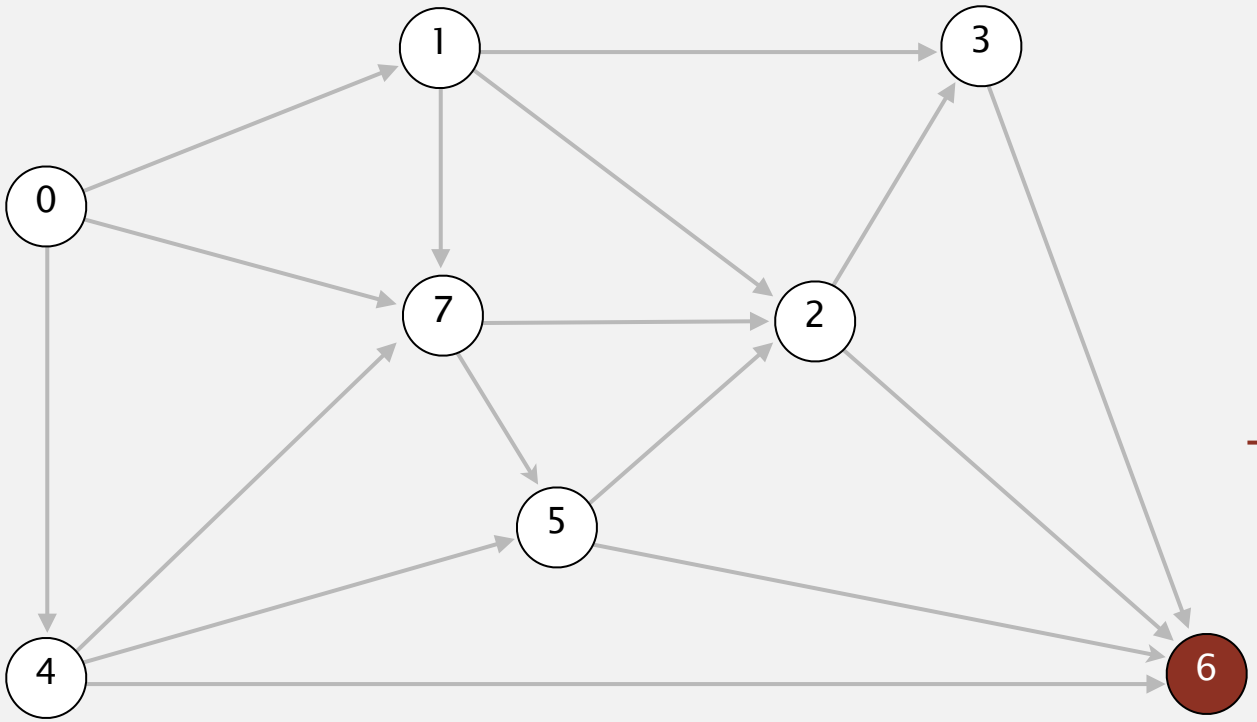
0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
→ 6	25.0	2→6
7	8.0	0→7

select vertex 6

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



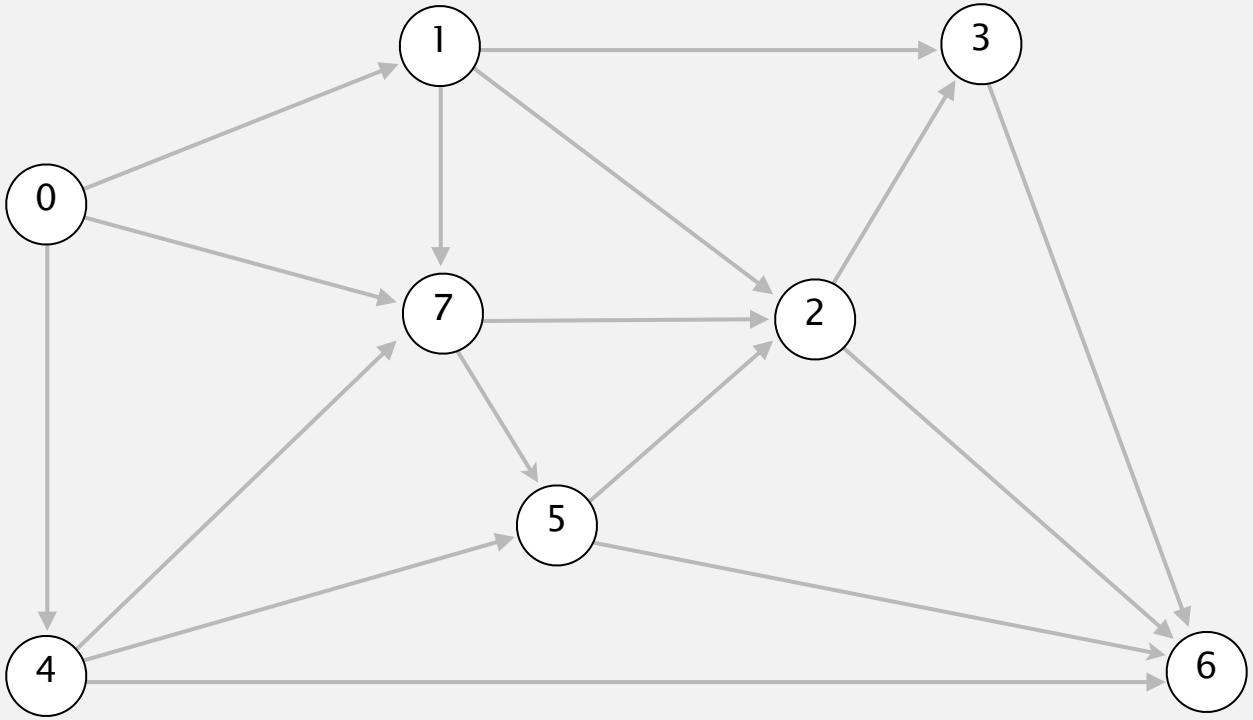
0 1 4 7 5 2 3 6 ↓

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
→ 6	25.0	2→6
7	8.0	0→7

relax all edges incident from 6

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.

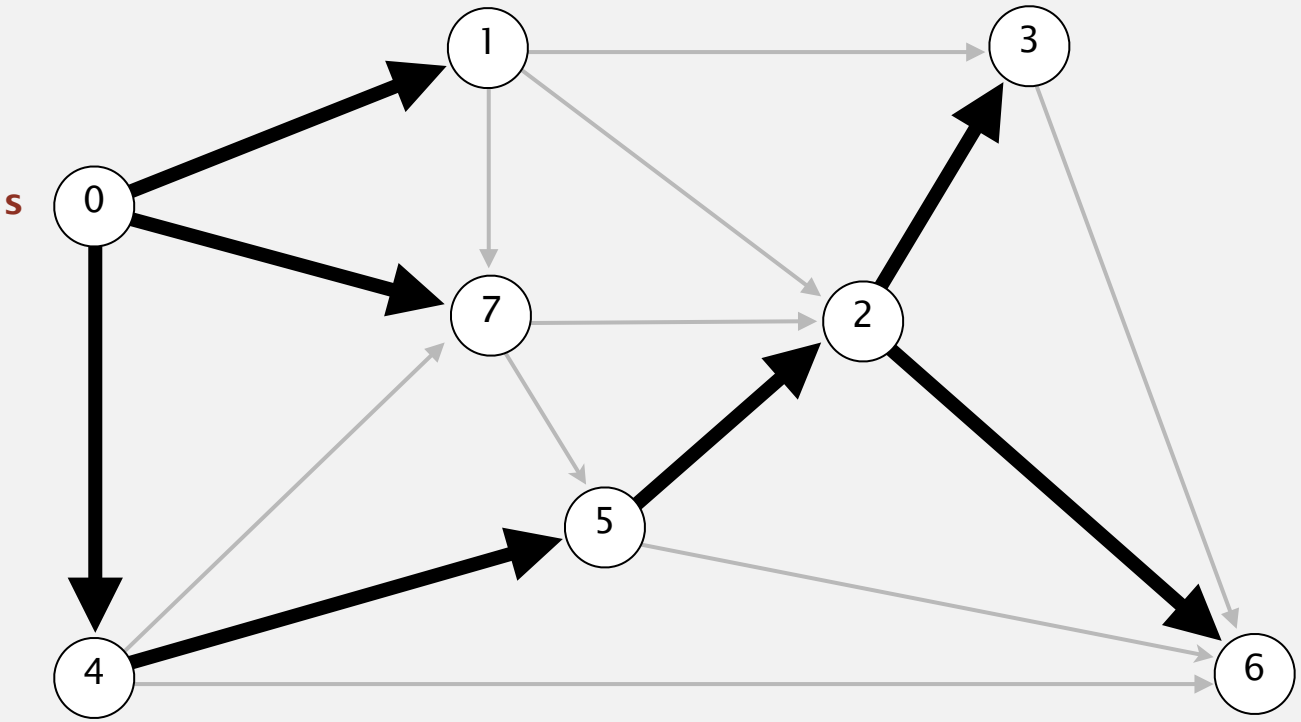


0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0	2→6
7	8.0	0→7

Topological sort algorithm

- Consider vertices in topological order.
- Relax all edges incident from that vertex.



shortest-paths tree from vertex s

0 1 4 7 5 2 3 6

v	distTo[]	edgeTo[]
0	0.0	-
1	5.0	0→1
2	14.0	5→2
3	17.0	2→3
4	9.0	0→4
5	13.0	4→5
6	25.0	2→6
7	8.0	0→7